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THE THERMAL LAMINAR BOUNDARY LAYER ON A CONTINUOUS CYLINDER

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Abstract—The thermal laminar boundary layer on a continuous isothermal cylinder for $Pr \gtrless 1$ has been investigated by approximate Kármán–Pohlhausen integral technique and exactly for small values of the curvature parameter X. From the comparison of both solutions the error of the approximate method has been estimated.

NOMENCLATURE

a, thermal diffusivity;
a_1, a_2, a_3, coefficients in series expansion (37);
A, parameter in boundary-layer velocity profile, equation (7);
B, parameter in boundary-layer temperature profile, equation (14);
c, specific heat;
$f(\xi, \eta)$, dimensionless stream function, equation (21);
$g(\xi, \eta)$, dimensionless temperature, equation (22);
h, h_t, radial coordinate of the outer edge of the momentum and thermal boundary layers, respectively;
k, thermal conductivity;
$Nu_R, \overline{Nu_R}$, local and average Nusselt number, equations (14) and (41), respectively;
Pr, Prandtl number, $Pr = \nu/a$;
q_R, rate of heat transfer per unit area;
r, radial coordinate originating on the axis of the cylinder;
R, radius of the cylinder;
t, temperature of the fluid;
t_R, surface temperature of the cylinder;
t_0, ambient temperature of the fluid;
u, axial fluid velocity component;
u_R, speed of the cylinder;
v, radial fluid velocity component;
x, axial coordinate;
X, curvature parameter, equation (11).

Greek symbols

η, transformed r-coordinate, equation (20);
μ, dynamic viscosity;
ν, kinematic viscosity;
ξ, transformed x-coordinate, equations (19) and (11);
ψ, stream function, equation (18).

1. INTRODUCTION

THE THERMAL laminar boundary layer on a continuous cylinder travelling through quiet ambient fluid is both of practical and theoretical interest and has been investigated by several authors.

The solution of thermal boundary-layer equations on a continuous cylinder for $Pr \leqslant 1$ has been obtained by the approximate Kármán–Pohlhausen integral technique by Pechoč [1] and Bourne and Ellistone [2], for $Pr \gtrless 1$ by Rotte and Beek [3]; an exact solution for the continuous flat sheet and for the continuous cylinder has been presented by Tsou *et al.* [4] and Gampert [5, 6] respectively.

The object of the present paper is to give a general solution of the boundary-layer equations on a continuous cylinder for $Pr \gtrless 1$ both by the Kármán–Pohlhausen method and exactly for small values of the curvature parameter X, making thus the error estimation of the approximate method possible. Simultaneously, the previous published solutions are given precision.

2. FORMULATION OF THE PROBLEM

The laminar boundary layer on a continuous cylinder with constant radius R, velocity u_R and temperature t_R, that moves axially through a stationary incompressible fluid with constant physical properties and temperature t_0, neglecting free convection and viscous dissipation, may be described by momentum, energy and continuity equations:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \quad (1)$$

$$u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial r} = a\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right) \quad (2)$$

$$r\frac{\partial u}{\partial x} + \frac{\partial (vr)}{\partial r} = 0 \quad (3)$$

with the boundary conditions

$$r = R: \quad u = u_R; \quad v = 0; \quad t = t_R \quad (4)$$

$r \to \infty$ or

$$x = 0,\ r > R:\quad u = v = 0;\ \partial u/\partial r = 0;\quad t = t_0;\ \partial t/\partial r = 0 \tag{5}$$

where x and r are coordinates along the axis of the cylinder and perpendicular to it, u and v are velocity components in the direction of x and r respectively, $t(x, r)$—temperature of the fluid in the boundary layer.

2.1. *Approximate solution*

Solution of the momentum boundary-layer equation (1) by Kármán–Pohlhausen technique has been obtained by several authors [1, 2, 7] by using the logarithmic velocity profile

$$u/u_R = 1-(1/A)\ln(r/R)\quad \text{for } r \leqslant h;\quad u/u_R = 0 \text{ for } r \geqslant h \tag{6}$$

with dimensionless parameter A defined as

$$1/A = -(R/u_R)(\partial u/\partial r)_R \tag{7}$$

and

$$h = R\exp(A) \tag{8}$$

is the coordinate of the outer edge of the momentum boundary layer. For the parameter A the differential equation

$$\mathrm{d}A/\mathrm{d}X = 2A^2/[(A-1)\exp(2A)+A+1] \tag{9}$$

has been obtained, the solution of which gives [8]

$$X = \sum_{n=1}^{\infty} 2^n A^{n+1} n/(n+1)(n+2)! \tag{10}$$

where the curvature parameter X is defined as

$$X = \nu x/u_R R^2. \tag{11}$$

Integrating the energy equation (2) combined with the continuity equation (3) we obtain an integral equation

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_R^{\infty} u(t-t_0)r\,\mathrm{d}r = -aR[\partial(t-t_0)/\partial r]_R. \tag{12}$$

For the solution of this equation a logarithmic temperature profile has been used [1, 2]

$$\frac{t-t_0}{t_R-t_0} = 1-(1/B)\ln(r/R) \text{ for } r \leqslant h_t \tag{13a}$$

and

$$\frac{t-t_0}{t_R-t_0} = 0 \text{ for } r \geqslant h_t \tag{13b}$$

where the dimensionless parameter B is defined as

$$1/B = -R[\partial(t-t_0)/\partial r]_R/(t_R-t_0) = q_R/k(t_R-t_0) = Nu_R \tag{14}$$

and

$$h_t = R\exp(B) \tag{15}$$

is the coordinate of the outer edge of the thermal boundary layer.

The solution of the equation (12) depends on the value of Pr. For $Pr < 1$, the momentum boundary layer is thinner than the thermal boundary layer and it follows from (6) that the integration of equation (12) must be finished at the coordinate h. For $Pr > 1$ the thermal boundary layer is thinner and the integration must proceed only to the coordinate h_t.

Let us generally define the outer coordinate m of the boundary layer as

$$m = R\exp(K) \tag{16}$$

where $K = A$ for $Pr \leqslant 1$, $K = B$ for $Pr \geqslant 1$ and evidently $K = A = B$ for $Pr = 1$. Substituting the velocity profile (6) and the temperature profile (13) in the equation (12), integrating and eliminating the curvature parameter X with the help of (9) we obtain the differential equation

$$\frac{\mathrm{d}B}{\mathrm{d}A} = \frac{B/A}{e^{2K}(2AK-A-2K^2+2K-1)+A+1} \times \left\{\frac{2}{Pr}[e^{2A}(A-1)+A+1] - e^{2K}(2BK-B-2K^2+2K-1)-B-1\right\} \tag{17}$$

that presents a general solution of thermal boundary layer for any Pr.

2.2. *Exact solution*

The familiar transformation, i.e. introducing the stream function ψ as

$$\partial\psi/\partial x = -rv;\quad \partial\psi/\partial r = ru \tag{18}$$

the dimensionless coordinates ξ and η to replace x and r as

$$\xi = 4(X)^{1/2} \tag{19}$$

$$\eta = (u_R/\nu x)^{1/2}(r^2-R^2)/4R \tag{20}$$

the dimensionless function $f(\xi, \eta)$ instead of the stream function ψ as

$$f(\xi, \eta) = \psi/(\nu x u_R R^2)^{1/2} \tag{21}$$

and the dimensionless function $g(\xi, \eta)$ instead of the temperature t

$$(t-t_0)/(t_R-t_0) = g(\xi, \eta) \tag{22}$$

into the equations (1)–(3) yields two partial differential equations

$$\frac{\partial}{\partial\eta}\left[(1+\xi\eta)\frac{\partial^2 f}{\partial\eta^2}\right] + f\frac{\partial^2 f}{\partial\eta^2} + \xi\left[\frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2} - \frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta}\right] = 0 \tag{23}$$

$$\frac{1}{Pr}\frac{\partial^2 g}{\partial\eta^2}(1+\xi\eta) + \xi\frac{\partial g}{\partial\eta}\frac{1}{Pr} + f\frac{\partial g}{\partial\eta} + \xi\left[\frac{\partial f}{\partial\xi}\frac{\partial g}{\partial\eta} - \frac{\partial f}{\partial\eta}\frac{\partial g}{\partial\xi}\right] = 0 \tag{24}$$

where a simplified notation f and g is used for the functions $f(\xi, \eta)$ and $g(\xi, \eta)$ respectively.

The boundary conditions in the new coordinates are

$$f(\xi, 0) = 0;\quad (\partial f/\partial\eta)_{\eta=0} = 2;\quad g(\xi, 0) = 1 \tag{25}$$

$$(\partial f/\partial\eta)_{\eta\to\infty} = 0;\quad g(\xi, \eta)_{\eta\to\infty} = 0. \tag{26}$$

In region of small values of ξ one can use power expansion for the functions $f(\xi, \eta)$ and $g(\xi, \eta)$:

$$f(\xi, \eta) = \sum_{i=0}^{\infty} \xi^i f_i(\eta) \tag{27}$$

$$g(\xi, \eta) = \sum_{i=0}^{\infty} \xi^i g_i(\eta). \tag{28}$$

Taking only three terms of these series which converge rapidly for small ξ and substituting them into (23) and (24), a system of ordinary differential equations results:

$$f_0''' + f_0 f_0'' = 0 \quad (29)$$

$$f_1''' + f_0 f_1'' - f_0' f_1' + 2f_0'' f_1 + \eta f_0''' + f_0'' = 0 \quad (30)$$

$$f_2''' + f_0 f_2'' - 2f_0' f_2' + 3f_0'' f_2 + \eta f_1''' + f_1''(1+2f_1) - (f_1')^2 = 0 \quad (31)$$

$$g_0'' + Pr \cdot f_0 g_0' = 0 \quad (32)$$

$$g_1'' + Pr(f_0 g_1' - f_0' g_1 + 2f_1 g_0') + g_0' + \eta g_0'' = 0 \quad (33)$$

$$g_2'' + Pr(f_0 g_2' - 2f_0' g_2 + 2f_1 g_1' - f_1' g_1 + 3f_2 g_0') + g_1' + \eta g_1'' = 0 \quad (34)$$

with boundary conditions

$$\eta = 0:\ f_0' = 2;\ f_0 = 0;\ g_0 = 1$$

$$f_i' = 0;\ f_i = 0;\ g_i = 0 \text{ for } i > 0 \quad (35)$$

$$\eta \to \infty:\ f_i' = 0;\ g_i = 0. \quad (36)$$

3. NUMERICAL SOLUTION AND RESULTS

3.1. *Solution of the equation* (17)

The only known point $A = B = 0$ does not allow the solution of (17), as the expression for dB/dA is of an indetermined form. Therefore, a power series expansion

$$B = a_1 A + a_2 A^2 + a_3 A^3 + \ldots \quad (37)$$

was used to obtain the starting point for numerical integration. By substituting this series in (17) and comparing coefficients of the identical powers of A the equations for computation of a_i are found (Appendix 1). The values of a_i and the values of B, calculated for $A = 0.01$ from (37), are summarized in Table 1.

Table 1. The coefficients a_i in series expansion (37)

Pr	a_1	a_2	a_3	B
0.7	1.285714	−0.075630	0.004818	0.012850
1	1	0	0	0.01
10	0.270701	0.063478	0.010745	0.002713

Starting at these points, the differential equation (17) was then integrated by the fourth order Runge–Kutta method with the help of the calculator HP 9100. The calculated values of B for various values of X, for which the values of A were determined from (10), are displayed in Table 2. The values of B for $A < 0.01$ have been obtained directly from (37).

The values of Nu_R for $Pr = 1$, i.e. when $A = B$, have been published already [2] and are displayed for completeness and comparison only.

3.2. *Solution of equations* (29)–(34)

Equations (29)–(34) have been solved by the Taylor expansion in the form

$$y_{i,n+1}^{(m)} = \sum_{k=0}^{p} \frac{h^k y_{i,n}^{(k+m)}}{k!} \quad (38)$$

where the exponent in brackets means the order of the derivative for $y = f(\eta)$ and $g(\eta)$. The value of m is $m = 0$–2 and $m = 0$ and 1 for $f(\eta)$ and $g(\eta)$, respectively.

Table 2. Values of $Nu_R = 1/B$

$\log X$	A	Nu_R $Pr = 0.7$	$Pr = 1$	$Pr = 10$
−5	0.0077260	100.72	129.43	477.27
−4.5	0.013712	56.767	72.929	268.52
−4	0.024297	32.057	41.157	151.17
−3.5	0.042937	18.160	23.290	85.171
−3	0.075517	10.345	13.242	48.056
−2.5	0.13174	5.9497	7.5909	27.183
−2	0.22666	3.4771	4.4118	15.445
−1.5	0.38137	2.0849	2.6221	8.8437
−1	0.62020	1.2992	1.6124	5.1303
−0.5	0.96210	0.85281	1.0394	3.0401
0	1.4088	0.59518	0.70983	1.8611
0.5	1.9408	0.44192	0.51525	1.1923
1	2.5278	0.34648	0.39559	0.80773
1.5	3.1430	0.28369	0.31816	0.58078
2	3.7692	0.24005	0.26531	0.44141
2.5	4.3975	0.20819	0.22740	0.35151
3	5.0239	0.18397	0.19905	0.29045
3.5	5.6473	0.16494	0.17708	0.24695
4	6.2672	0.14958	0.15956	0.21464
4.5	6.8839	0.13691	0.14527	0.18979
5	7.4976	0.12627	0.13338	0.17012
5.5	8.1087	0.11721	0.12332	0.15418
6	8.7174	0.10940	0.11471	0.14100

The step of integration $h = 0.01$ and $p = 5$ was used for the numerical calculation. The higher derivatives of f_i and g_i are defined with the help of (29)–(34) and substituted in (38), so that the values of f_i and g_i in the step $(n+1)$ are given only by the values of the function, its first derivative and for f_i its second derivative, too, in the step n. The values of f_i, f_i' and g_i for $\eta = 0$ are known from the boundary condition (35). The values $f_i''(0)$ and $g_i'(0)$ have been approximated by trial and error so as to satisfy to a maximum accuracy possible with the used calculator HP 9100 the boundary condition (36). The accuracy of the solution was tested by repeated calculation with the integration step halved.

The approximated values of $f_i''(0)$ and the values of f_i, f_i' and f_i'' for $\eta \leq 18.4$ have been published in previous work [8]. The values of $g_i'(0)$ for various values of Pr are displayed in Table 3. The maximum error, Δ, of the calculated values of $g_i(0)$ is $\Delta = 5 \cdot 10^{i-9}$.*

From the definition of the local rate of heat transfer per unit area of the cylinder surface

$$q_R = -k\left[\frac{\partial(t - t_0)}{\partial r}\right]_R \quad (39)$$

using equations (19), (22), (28) and rearranging we obtain for the local Nusselt number

$$Nu_R X^{1/2} = -\sum_{i=0}^{2} 2^{2i-1} X^{i/2} g_i'(0). \quad (40)$$

The equation for average Nusselt number $\overline{Nu}_R$

$$\overline{Nu}_R = \frac{1}{X}\int_0^X Nu_R \, dX \quad (41)$$

*Detailed tables of $g_i^{(m)}$ for $0 \leq i \leq 2$, the order of derivative $m = 0$ and 1, and for $\eta \leq 21$ have been displayed in the report [9] and will be made available upon request by the authors.

Table 3. Approximated values of $g_i'(0)$

i	$g_i'(0)$		
	Pr = 0.7	$Pr = 1$	$Pr = 10$
0	−0.69847170	−0.88749662	−3.36058656
1	−0.1823863	−0.1900993	−0.2013562
2	0.009027	0.009270	0.001843

becomes by substituting equation (40)

$$\overline{Nu_R X^{1/2}} = -\sum_{i=0}^{2} \frac{2^{2i}}{i+1} X^{i/2} g_i'(0). \qquad (42)$$

Using the values $g_i'(0)$ from Table 3 with the equations (40) and (42), we obtain equations for the calculation of the local and average Nusselt numbers. In view of the used expansion the applicability of these equations is restricted to $X \leqslant 0.06$ for $Pr = 0.7$, $X \leqslant 0.07$ for $Pr = 1$ and $X \leqslant 1.4$ for $Pr = 10$, when the value of the third term of the series is approximately 5% of the second term and 1% of the sum of first and second term.

The approximate solution of the thermal laminar boundary layer on a continuous cylinder for $Pr \lesssim 1$ by Rotte and Beek [3] starts from their equations (4a) and (4b) but the last term on the left side of the equation (4a) for $Pr \geqslant 1$ has an incorrect negative sign. As there have been provided no numerical results of the solution of these equations in the paper, it is difficult to check the solution only from the plots of various combinations of dimensionless groups published there.

5. CONCLUSION

The demonstrated solution of the thermal boundary layer on a continuous cylinder enables to obtain the local and average Nusselt numbers for $Pr \lesssim 1$. By comparison with an exact solution for small values of X, the estimation of the error of the approximate Kármán–Pohlhausen solution has been made possible. The values of $Nu_R X^{0.5}$ obtained both by the approximate and exact solutions [equations (17) and (40)] have been displayed in Table 4. As it is evident, the approximate solution underestimates the rate of heat transfer (Nusselt number) by 8–10% in the range of Pr and X displayed in the table. Similarly as at the momentum boundary layer [8], the error has for $Pr \leqslant 1$ a slightly increasing trend with increasing X, and only for $Pr = 10$ the expected (and supposed by Bourne and Elliston [2]) slight decrease of the error with increasing X has been obtained.

For practical calculations, it can be supposed that the approximate solution underestimates Nusselt number by approximately 10% in the whole range of X, what coincides with recent results of Gampert [5].

Table 4. Values of $Nu_R X^{1/2}$ for small X

X	Pr = 0.7			$Pr = 1$			$Pr = 10$		
	$Nu_R X^{1/2}$		Δ%	$Nu_R X^{1/2}$		Δ%	$Nu_R X^{1/2}$		Δ%
	(17)	(40)		(17)	(40)		(17)	(40)	
0.0001	0.32057	0.35288	−9.2	0.41157	0.44754	−8.0	1.51151	1.68432	−10.3
0.0005	0.32433	0.35736	−9.2	0.41568	0.45221	−8.1	1.51635	1.68929	−10.2
0.001	0.32714	0.36070	−9.3	0.41875	0.45570	−8.1	1.51959	1.69301	−10.2
0.005	0.33893	0.37467	−9.5	0.43162	0.47026	−8.2	1.53385	1.70870	−10.2
0.01	0.34771	0.38499	−9.7	0.44118	0.48103	−8.3	1.54464	1.72042	−10.2
0.04	0.37725	0.41930	−10.0	0.47336	0.51682	−8.4	1.58065	1.76025	−10.2
0.05	0.38414	0.42719	−10.1	0.48085	0.52506	−8.4	1.58913	1.76960	−10.2
0.06	0.39034	0.43425	−10.1	0.48759	0.53243	−8.4	1.59680	1.77805	−10.2
0.07				0.49376	0.53915	−8.4	1.60387	1.78581	−10.2
0.1							1.62235	1.80617	−10.2
0.5							1.76018	1.95768	−10.1
1							1.86109	2.06826	−10.0
1.4							1.92323	2.13615	−10.0

4. COMPARISON WITH PREVIOUS RESULTS

Tsou *et al.* [4] have published the results of an exact solution of heat transfer through boundary layer on a continuous flat sheet in terms of $Nu_x/(Re_x Pr)^{1/2}$. Multiplying these values by $2Pr^{0.5}$ (with respect to the substitution used), we obtain results which differ in one unity on the fourth place from the values of $g_0'(0)$ in Table 3.

The values of $g_i'(0)$ for $Pr = 0.7$ published by Gampert [5, 6] are $g_0'(0) = -0.698583$; $g_1'(0) = -0.184626$; $g_2'(0) = -0.0226193$. The mistakes in the second and third equation in the equation system (1.19) [5] and (6) [6] (omission of the multiplication of f_0''' in the second and f_1''' in the third equation by η) are obviously due to misprint. Nevertheless, it is evident from the published values of the functions f_i and g_i [5, 6] that the fulfilment of the boundary condition (36) is strictly imposed for $\eta = 8$ (an abrupt change of values). From the values of $g_i'(8)$ it follows that the error of $g_i'(0)$ is of the order 10^{i-4} for $i = 0, 1, 2$.

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APPENDIX 1

The coefficients a_1, a_2 and a_3 for $Pr \leqslant 1$, i.e. when $K = A$, have been obtained by power series expansion by Bourne and Elliston [2]. For $Pr \geqslant 1$ is $K = B$ and we obtain from (17):

$$a_1^3 - 3a_1^2 + 2Pr^{-1} = 0 \tag{43}$$

$$a_2 = a_1(-2a_1^2 + 5a_1 - 3)/(5a_1 - 9) \tag{44}$$

$$a_3 = [5a_2^2(7a_1 - 6) + 5a_1^2 a_2(11a_1 - 16) + 3a_1^2(a_1 - 1)(3a_1^2 - 2a_1 - 3)]/5a_1(12 - 7a_1). \tag{45}$$

Equation (43) has three real roots. The right root can be chosen from the condition $0 < a_1 \leqslant 1$ as $0 < B \leqslant A$ for $Pr \geqslant 1$ and is defined by equation

$$a_1 = 1 - 2\cos\left(\frac{\pi + \varphi}{3}\right) \tag{46}$$

with

$$\cos\varphi = 1 - 1/Pr. \tag{47}$$

COUCHE LIMITE THERMIQUE LAMINAIRE SUR UN CYLINDRE CONTINU

Résumé—La couche limite thermique laminaire sur un cylindre isotherme pour $Pr > 1$ a été étudiée de façon approchée par la technique de Karman–Polhausen et de façon exacte pour les petites valeurs du paramètre de courbure X. Par comparaison des deux solutions, on a estimé l'erreur sur la méthode approchée.

DIE LAMINARE THERMISCHE GRENZSCHICHT AN EINEM UNENDLICH AUSGEDEHNTEN ZYLINDER

Zusammenfassung—Für einen unendlich ausgedehnten, isothermen Zylinder wurde die laminare thermische Grenzschicht für $Pr \gtrless 1$ näherungsweise mit Hilfe der Karman-Pohlhausen-Integraltechnik gelöst. Für kleine Krümmungsparameter X wird eine exakte Lösung angegeben. Durch Vergleich beider Lösungen kann der Fehler der Näherungslösung abgeschätzt werden.

ТЕМПЕРАТУРНЫЙ ЛАМИНАРНЫЙ ПОГРАНИЧНЫЙ СЛОЙ НА НЕОГРАНИЧЕННОМ ЦИЛИНДРЕ

Аннотация — Температурный ламинарный пограничный слой на неограниченном изотермическом цилиндре при $Pr \lessgtr 1$ рассчитывается приближенно интегральным методом Кармана–Польгаузена и точно для небольших значений параметра кривизны X. Из сравнения обоих решений оценена погрешность приближенного метода.